Towards the AdS dual of SYK

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KITP

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This, in principal, fully determines the bulk Lagrangian: all the masses and couplings of the (infinite) tower of bulk fields.

However, we are far from a clear physical understanding of what the bulk theory actually is.

I will first remind you of some basic aspects of the canonical example of AdS/CFT: the duality between string theory in $AdS_5 \times S^5$ and $\mathcal{N}=4$ super Yang-Mills.

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One can compute the dimensions of these operators. At large ’t Hooft coupling, most of these operators acquire a large dimension, as one would expect.

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The ’t Hooft coupling is dual to the ratio of the AdS scale to the string scale, so this is what we would expect.

In SYK, the fundamental fields are fermions χ_i out of which we can construct O(N) singlets,

$$\mathcal{O}_n \sim \frac{1}{N}\sum_{i=1}^{N} \chi_i \partial_\tau^{1+2n} \chi_i$$

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The lowest dimension operator is h=2, and is special. It is responsible for the breaking of conformal invariance and maximal chaos, and is dual to the dilaton. It is roughly like the graviton (talks by Maldacena, Verlinde).

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But this of course is not the usual string theory, since there are far fewer fields.

The bulk action is of the form

$$S_{bulk} = \int d^2x \sqrt{g} \left[\frac{1}{2} (\partial\phi_n)^2 + \frac{1}{2} m_n^2 \phi_n^2 + \frac{1}{\sqrt{N}} \lambda_{nmk} \, \phi_n \phi_m \phi_k + \ldots \right]$$

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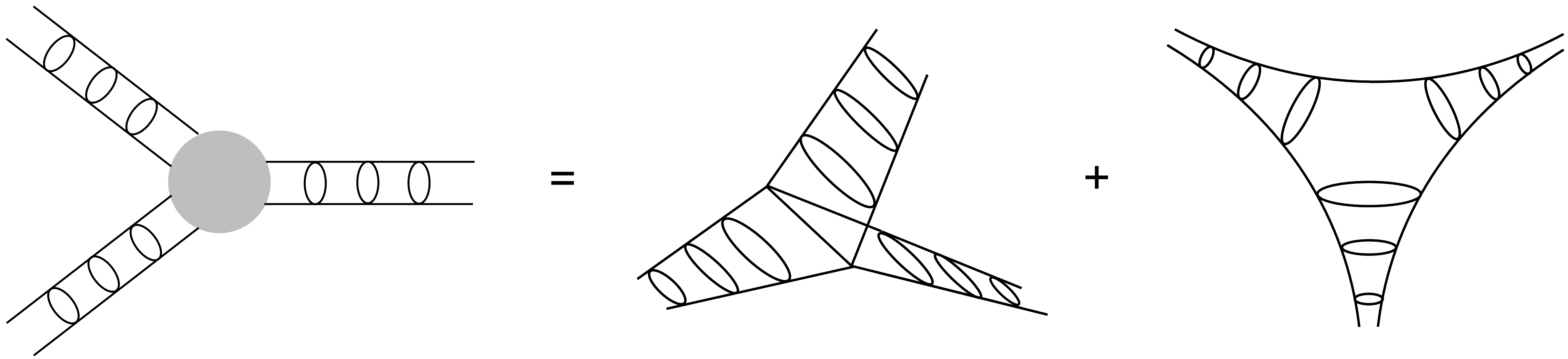
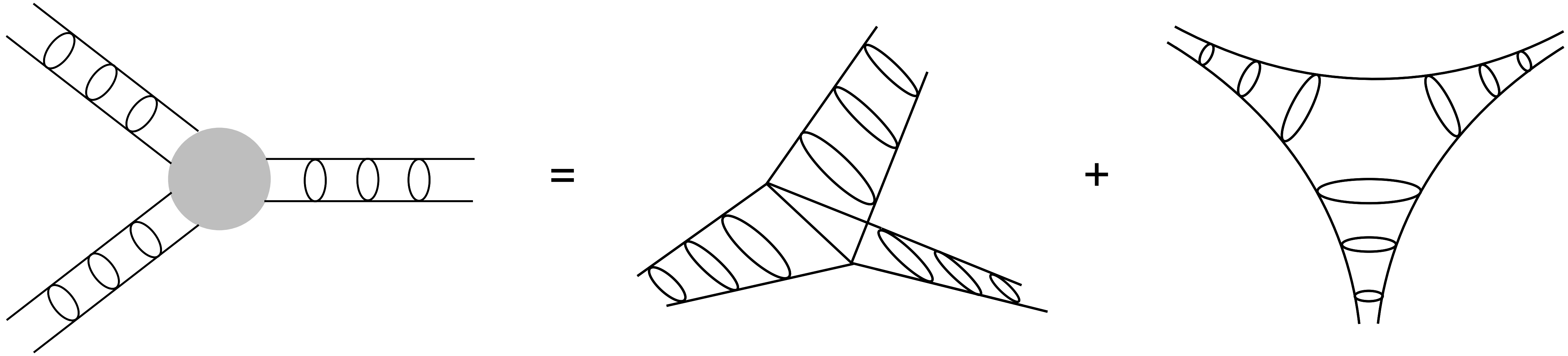
To find the cubic couplings, we must compute the SYK three-point function of bilinears (six-point function of fermions), for the quartic couplings we need the four-point function of bilinears, and so on.

I will now make some technical remarks abut the computation of all-point correlation functions in SYK



$$c_{h_1 h_2 h_3} \qquad\qquad \mathcal{O} \sim \chi_i \partial_\tau^{1+2n} \chi_i$$



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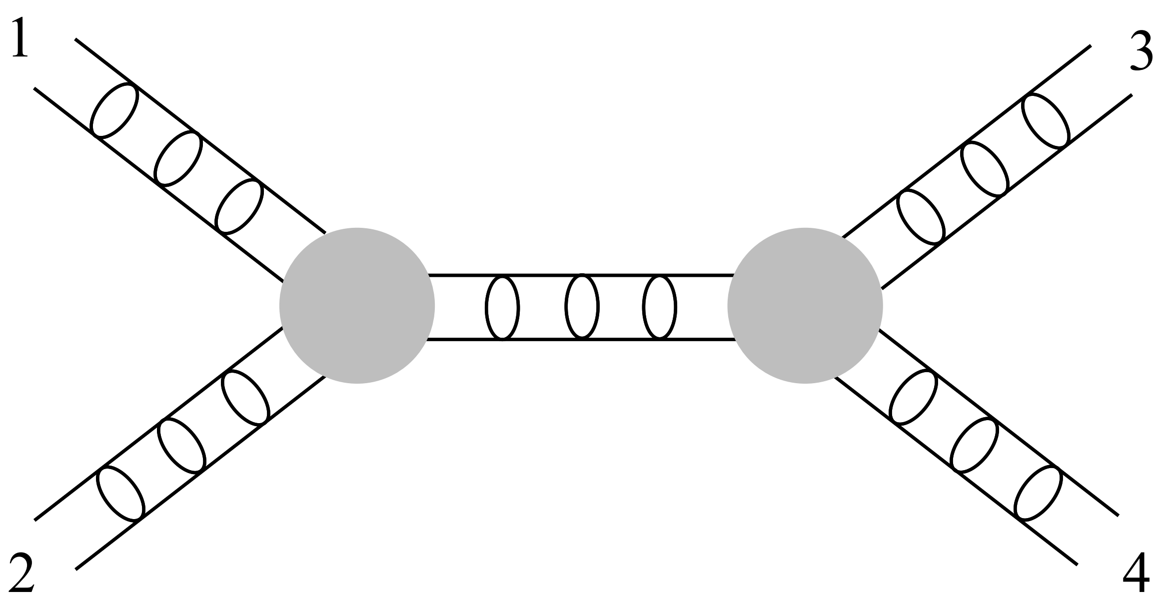
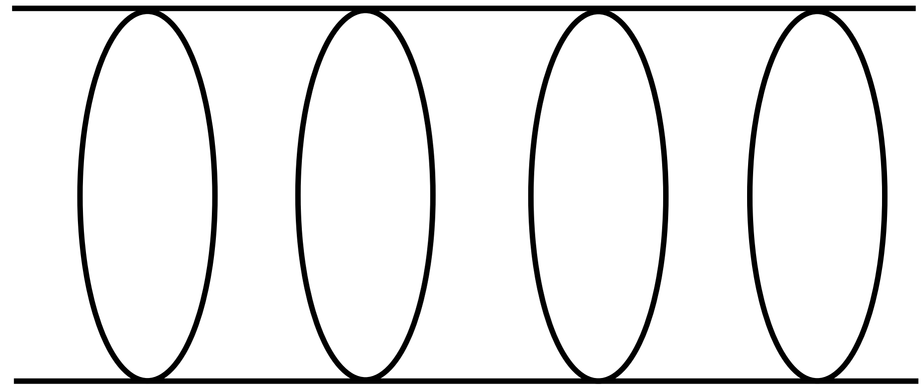


$$\int_{\mathcal{C}} \frac{dh}{2\pi i}\, \widetilde{\rho}(h)\, c_{h_1 h_2 h}\, c_{h_3 h_4 h}\, \mathcal{F}^h_{h_i}(\tau_i)$$

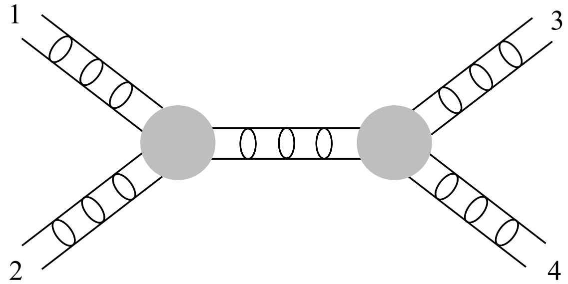


$$\int_{\mathcal{C}} \frac{dh}{2\pi i}\, \rho(h)\, \mathcal{F}^{h}_{\Delta}(\tau_i) \qquad\qquad \rho(h) \sim \frac{k(h)}{1-k(h)}$$

- h-space is to the conformal group, SL(2,R), what Fourier space is to translations

$$\int \frac{dp}{2\pi} f(p)\, e^{ipx}$$

Maldacena Stanford
Polchinski, V.R
Kitaev



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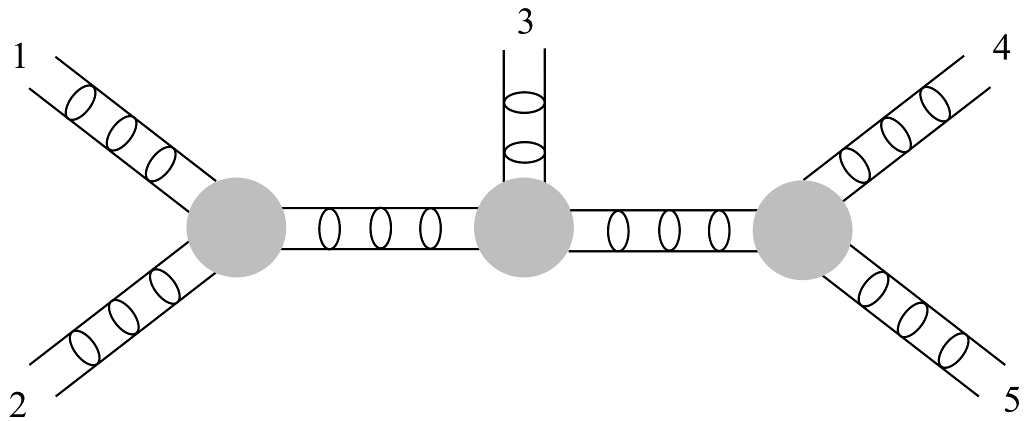
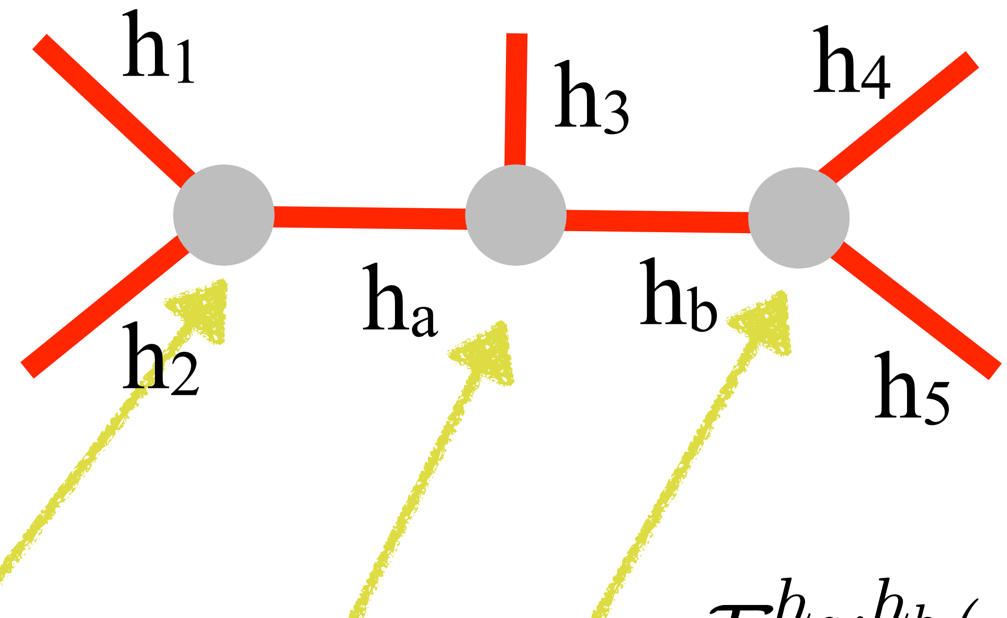
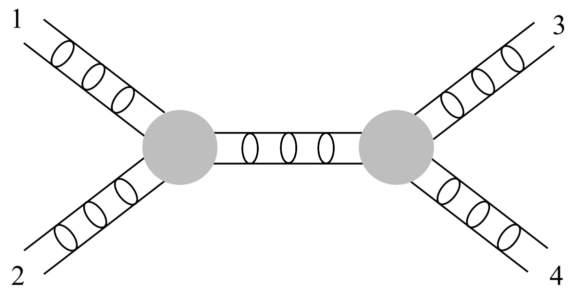




$$\int_{\mathcal{C}} \frac{dh_a}{2\pi i}\, \widetilde{\rho}(h_a)\, \int_{\mathcal{C}} \frac{dh_b}{2\pi i}\, \widetilde{\rho}(h_b)\, c_{h_1 h_2 h_a} c_{h_a h_3 h_b} c_{h_b h_4 h_5}\, \mathcal{F}^{h_a, h_b}_{h_i}(\tau_i)$$



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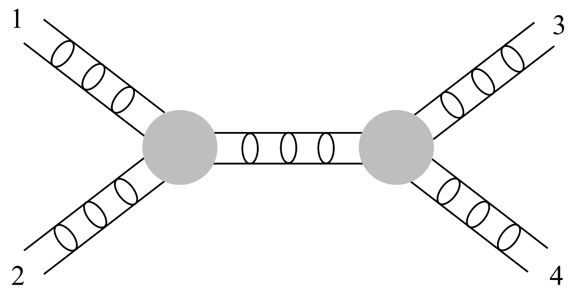
- These are simple rules for summing an infinite number of diagrams. It doesn't matter that the four-point function is made up of ladders. These apply to any four-point functions.



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- This is not just an OPE expansion. The $c_{h_1 h_2 h_3}$ are the analytically extended OPE coefficients of the single-trace operators. The four-point function is a sum of conformal blocks of single-trace operators and double-trace operators. This emerges upon closing the contour. Somehow, the analytically extended single-trace OPE coefficients encode the information about the double-trace OPE coefficients.

The only result for the correlation function that I will use in the rest of the talk is the three-point function of bilinears.

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Notice that if one sets $n_1=n_2=n_3$, this grows exponentially.

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Summary: The bulk has a tower of fields, with masses roughly spaced by even integers, and cubic couplings of the form on the previous slide.

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However, the cubic couplings seem qualitatively different from what one gets from a Kaluza-Klein reduction.

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This gives some combinatorial factors, vaguely like what we found.

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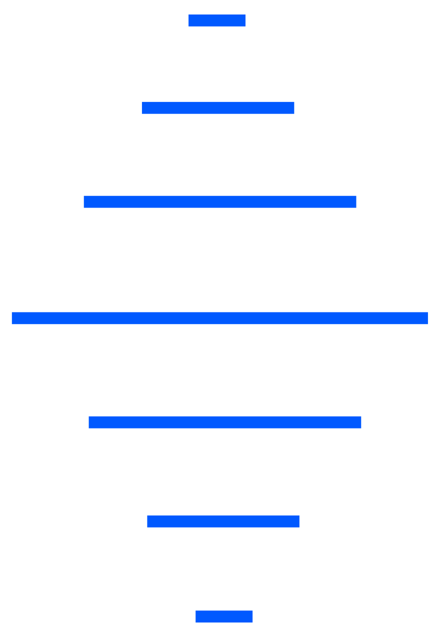
One can try to instead allow longitudinal motion: a string that is expanding and contracting.



Bardeen, Bars, Hanson, Peccei, '76

Maldacena, '05, Maldacena Stanford, '16

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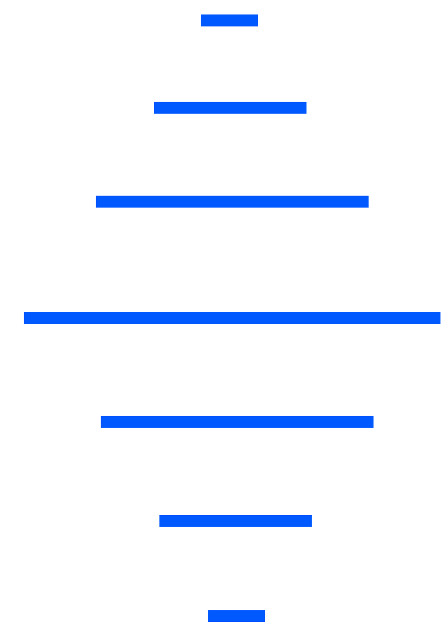

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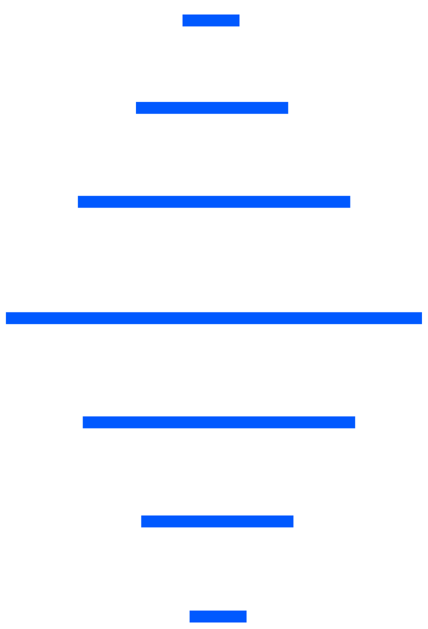

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To get the cubic couplings, we would need to quantize the string. It is unclear how to do this.

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We have not yet succeeded in solving the 't Hooft model in AdS, so I do not know the answer.

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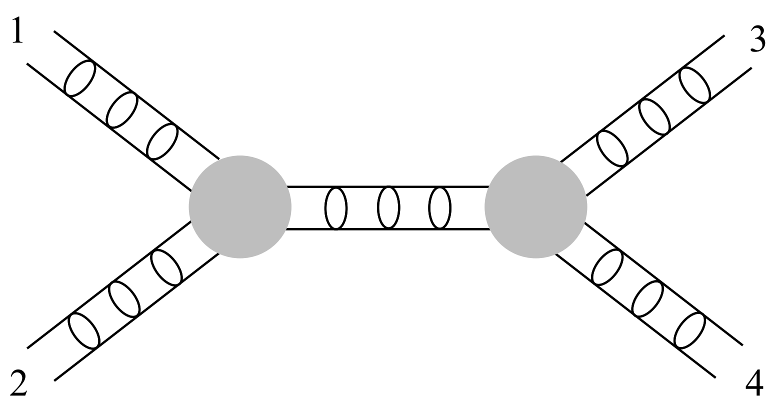
- The correlation functions take a simple form, when written correctly.



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- The cubic couplings, of the large mass fields, take a combinatorial form,

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- Some possibilities are a string with longitudinal motion, or 2d QCD with fermions. We do not yet know if this gives the correct cubic couplings.

- It is possible that one should be trying to find the dual not of SYK, but of tensor models. Tensor models have the same melonic Feynman diagrams as SYK, but their symmetry is O(N)×O(N)×O(N),

$$S = \int d\tau \left(\frac{1}{2} \psi_{abc} \partial_\tau \psi_{abc} + \frac{1}{4} g \, \psi_{abc} \psi_{ade} \psi_{fbe} \psi_{fdc} \right)$$

Gurau; Witten;
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- There are a huge number of singlets. This gives an extraordinary large number of bulk fields, far far more than SYK, and far more than string theory.

- What the dual of tensor models might be is a mystery.